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Inertia dominated forces on submarine pipelines near seabed

Forces exercées sur des conduites sous-marines près du fond dans le cas à inertie prépondérante

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SUMMARY
An experimental investigation on regular wave induced forces on a smooth submarine pipeline fixed horizontally near a simulated seabed is carried out in the inertia dominated regime. A simple potential flow around the cylinder is assumed in analysing the hydrodynamic forces on submarine pipelines considering the size of the pipe and wave conditions employed in this study. The inline hydrodynamic coefficient of inertia and transverse hydrodynamic coefficients of lift and vertical inertia are evaluated using the measured forces and through the use of least squares method. These inline and transverse hydrodynamic coefficients are correlated with the scattering parameter and gap ratio of the pipeline from the seabed. Further the effect of depth parameter on these hydrodynamic force coefficients is investigated.

RÉSUMÉ
L'article traite d'un recherche expérimentale sur les forces engendrées par une houle régulière sur une conduite sous-marine lisse fixée horizontalement à proximité d'un fond marin, dans le domaine des phénomènes à inertie prépondérante. On suppose un simple écoulement potentiel autour du cylindre en se basant sur la taille du tuyau et les caractéristiques de la houle considérée. Le coefficient hydrodynamique longitudinal d'inertie et les coefficients hydrodynamiques transversaux de portance et d'inertie verticale sont évalués à partir des forces mesurées en utilisant la méthode des moindres carrés. Ces coefficients, hydrodynamiques longitudinaux et transversaux sont corréles au paramètre de dispersion et à la distance relative du tuyau au fond. De plus, on a étudié l'influence du paramètre profondeur sur ces coefficients de la force hydrodynamique.

1 Introduction
Submarine pipelines offer an efficient mode of transportation of oil and natural gas from offshore production platforms to onshore terminal facilities. These pipelines are laid either directly on the
seabed, buried or placed at certain clearances from the sea floor depending on the bottom topography along their routes. When placed in a hostile environment such as the oceans, these pipelines are subjected to loadings arising from waves and currents. An accurate evaluation of wave and/or current induced forces on submarine pipelines is necessary to provide a compromise between economy and design.

The Morison equation (8) is the most widely used force model to evaluate the hydrodynamic forces due to waves on slender structural members. According to the Morison equation, the total instantaneous inline force, \( F \) is a linear combination of inertia and drag forces and is given as

\[
F(t) = F_i + F_D = 0.25C_m \rho \pi D^2 \dot{u}(t) + 0.5C_D \rho D u(t) | \dot{u}(t) |
\]

where \( F_i, F_D = \) inertial and drag forces per unit length respectively at time \( t \), \( \rho = \) mass density of water, \( D = \) diameter of the structural member, \( u, \dot{u} = \) instantaneous inline water particle velocity and acceleration respectively and \( C_m, C_D = \) hydrodynamic coefficients of inertia and drag respectively. The hydrodynamic coefficients have to be necessarily determined either from laboratory or prototype experiments. Considerable attention has been focussed by many researchers to correlate these hydrodynamic coefficients with some of the flow parameters and structural conditions.

It has been found by many investigators that when the Keulegan-Carpenter number or period parameter, \( K (= U_{\text{max}} T / D, U_{\text{max}} = \) amplitude of the inline water particle velocity and \( T = \) wave period) is very small the flow around the pipeline does not separate and no vortices will be generated from the cylinder since the flow will be reversed in every half cycle and the water particles will not have sufficient time to travel a long distance from the cylinder. The flow around the pipeline under such conditions can be considered to be potential flow. It is also reasonable to assume that the corresponding wave forces on the cylinder will be predominantly inertial. The wake dependent drag forces will be negligibly small compared to inertia forces and can therefore be conveniently ignored for all practical purposes.

Nath and Yamamoto [9] reported that when the relative water particle displacement, \( A/D < 0.4 \) (\( K < 2.5 \)) where \( A \) is the inline water particle displacement, and when the depth of submergence is a few cylinder diameters, then the potential flow calculations may be useful. In fact, it is reported that for a cylinder submerged as little as one diameter, the free surface effects are negligible. They also found that for a cylinder placed on the plane boundary, \( C_m \) is more than twice that for a cylinder in a free stream. From the laboratory experiments, Yuan Jen [18] found that in relatively deep water, \( d/L \geq 0.175 \) (\( d = \) still water depth and \( L = \) wave length) and low wave steepness, \( H/L < 0.02 \) (\( H = \) wave height) the total force was predominantly inertial.

Yamamoto et al. [16] found that the hydrodynamic coefficients can be properly evaluated using potential flow theory if the wake dependent drag forces are negligible. Grace [4] also discussed that when \( K < 4 \), the inertia forces may be considered to be more dominant than the drag forces. It has been reported by Yamamoto et al. [17] that when \( L > 10D \) and when the depth of submergence is more than about two cylinder diameters, the free surface effects can be ignored and the flow can be considered to be uniform for potential flow calculations. Nath et al. [10] found from their experiments that when \( A/D < 1.0 \), the wake will have no time to form and that the potential flow theory can predict well the transverse forces as a function of gap ratio of the pipeline, \( e/2a \) (\( a = \) radius of the pipeline).

Extensive data on the hydrodynamic coefficients for a horizontal circular cylinder have been provided by Sarpkaya [12, 13] for sinusoidally oscillating planar flow. He found that the transverse force is as large as the inline force and that its frequency depended on \( K \). The potential flow, the
cyclical transition from potential flow to wake flow and fully developed wake flows are classified by Wright and Yamamoto [14] in terms of $A/D$. Cheong et al. [1] and Jothi Shankar et al. [5] reported the results of their experimental investigations on submarine pipelines in the inertia-drag regime and with the pipe placed at various gaps from a plane boundary and subjected to regular and random waves. In both the cases, the inline and transverse hydrodynamic coefficients were found to depend on $K, e/2a$, where $a$ is the radius of the pipeline and depth parameter, $d/2a$. In addition Jothi Shankar et al. [5] concluded that the hydrodynamic coefficients obtained from random wave tests using significant wave concept were in good agreement with those from regular wave tests under similar pipeline conditions.

The primary objective of this paper is to present the results of an experimental investigation on regular wave induced forces on a smooth submarine pipeline model placed horizontally near a simulated seabed and with its axis parallel to the crests of the oncoming waves in the inertia dominated regime. The inline and transverse hydrodynamic forces are analysed based on the assumption that the flow around the pipeline is a simple potential flow around the cylinder. The inline hydrodynamic coefficient of inertia and transverse hydrodynamic coefficients of lift and vertical inertia are correlated with the scattering parameter, gap ratio of the pipeline from the seabed and depth parameter.

2 Theoretical considerations

2.1 Inline forces

Fig. 1 depicts the definition sketch of the submarine pipeline of diameter, $D$, fixed horizontally at an elevation or gap of $e$ from a plane boundary and in a constant water depth of $d$. It was discussed in the previous section that equation (1) can be used to predict the wave induced forces when the pipeline is situated in the inertia-drag regime [1, 12]. However when the wave and pipeline conditions are such that the pipeline is in the inertia dominated regime, the total inline force on the pipeline will be predominantly inertia and the drag forces due to viscous effects are negligible compared to inertia forces. The instantaneous inline force per unit length of the pipeline, $F_{II}$ in

![Fig. 1. Definition sketch.](image)
this regime of flow is given by the inertial component of the Morison equation and using linear wave theory it is expressed as,

\[ F_{H}(t) = 0.25C_{m} \varrho \pi D^{2} \dot{u}(t) \]

or

\[ F_{H}(\theta) = 0.25C_{m} \varrho \pi D^{2}(- U_{\text{max}} \sigma \sin \sigma t) \]  

or

\[ F_{H}(\theta) = 0.25C_{m} \varrho \pi D^{2}(- U_{\text{max}} \sigma \sin \theta) \]  

in which \( \sigma \) = wave angular frequency \( (= 2\pi/T) \), \( \theta \) = wave phase angle \( (- \sigma t) \) and other parameters have been defined earlier. Equation (3) implies that \( F_{H} \) is maximum when \( \dot{u} \) is maximum. However, in order to account for the occurrence of the maximum inline force, \( F_{H_{\text{max}}} \) relative to the occurrence of \( \dot{u}_{\text{max}} \) in a wave cycle, equation (3) is slightly modified as,

\[ F_{H}(\theta) = -0.25C_{m} \varrho \pi D^{2} U_{\text{max}} \sigma \sin (\theta - \delta) \]  

in which \( \delta \) = phase difference between \( F_{H_{\text{max}}} \) and \( \dot{u}_{\text{max}} \) in a cycle.

2.2 Transverse forces

Many investigators, including the authors [for example 1, 5, 6, 12, 13] have reported that for a pipeline in the inertia-drag regime, the transverse force phenomenon is quite complex because of the complicated nature of the flow near a plane boundary, boundary layer growth on the boundary and pipe, reversal of flow in every half cycle, generation and shedding of vortices from the pipe which are washed back and forth over the cylinder during every flow reversal. It is also found, in addition, that there exist several frequency components in the transverse forces. All these complexities make it impossible to have a simple transverse force model like the Morison equation for the inline force. However, for a pipeline in the inertia dominated regime, as in the present study, the total instantaneous transverse force is a linear combination of lift and vertical inertia forces. The presence of the pipe near the seabed causes increased velocity between the pipe and boundary and causes asymmetrical pressure distribution around the pipe. The lift component of the transverse force is due to this asymmetrical pressure distribution. The total instantaneous transverse force per unit length of the pipe, \( F_{v} \) is thus given as,

\[ F_{v}(\theta) = F_{L} + F_{vi} = 0.5C_{L} \varrho D u^{2}(\theta) + 0.25C_{m} \varrho \pi D^{2} \dot{w}(\theta) \]  

in which, \( F_{L} \), \( F_{vi} \) = transverse lift and inertia forces respectively, \( \dot{w} \) = instantaneous transverse water particle acceleration and \( C_{L} \) and \( C_{m} \) = transverse hydrodynamic coefficients of lift and vertical inertia respectively. In the above formulation, it is assumed that the vertical drag forces are negligible.

2.3 Potential flow theory

A circular cylinder moving parallel or normal to a wall can be expressed as an infinite series of properly distributed doublets [15, 16]. The complex potential and the forces that act on the cylinder are derived using the method of images and Blasius theorem respectively. The details can be found in [7, 15]. The inertia coefficient, \( C_{m} \) and lift coefficient, \( C_{L} \) are evaluated as,

\[ C_{m} = 2 \left( 1 + \sum_{j=1}^{\infty} m_{j} \right) \]  

\[ F_{H}(\theta) = -0.25C_{m} \varrho \pi D^{2} U_{\text{max}} \sigma \sin (\theta - \delta) \]  

in which \( \delta \) = phase difference between \( F_{H_{\text{max}}} \) and \( \dot{u}_{\text{max}} \) in a cycle.
\[ C_L = -4\pi \sum_{i=0}^{\infty} \sum_{k=0}^{\infty} \frac{m_i m_k}{\left( \frac{s}{a} - q_i - q_k \right)^3}; \ s > a \] (7)

where

\[ m_n = q_1^2 q_2^2 q_3^2 \ldots q_n^2; \ m_0 = 1 \] (8)

and

\[ q_n = \frac{1}{\left( \frac{s}{a} - q_{n-1} \right)^2}; \ q_0 = 0 \] (9)

in which \( s \) = distance of the cylinder centre from the bottom boundary and \( m \) and \( q \) = parameters related to the strength and location of doublets respectively. Potential flow theory thus predicts that the inertia coefficient, \( C_m \) for a cylinder moving either parallel or normal to a plane boundary is the same in both the directions.

3 Experimental set-up and procedure

A detailed experimental investigation on wave induced forces on a smooth submarine pipeline was carried out in a well controlled wave tank test facility at the Hydraulic Engineering Laboratory, National University of Singapore. The details of the wave tank along with the experimental arrangements for the investigation of wave induced forces on submarine pipeline are reported by Jothi Shankar et al. [1, 6]. The wave tank measures 36 m in length, 1.3 m height and a uniform width of 2 m. The paddle type wave generating system installed at one end of the wave tank generates regular and random waves depending on the input signal. A long beach at the other end absorbs the wave energy so that wave reflection is negligible. A 1 m long wire mesh filter placed about 1.5 m from the wave board filters out any secondary reflections from the wave board itself. The test section was chosen near the middle of the wave tank, which is about 16 m from the wave board. A smooth transition provided both on upstream and downstream sides of the test section for about 1.5 m length along the side walls ensures smooth entry of waves into the test section. A smooth perspex cylinder of 150 mm OD and 4 mm thick was chosen for the present model investigations. The total instantaneous inline and transverse forces on the cylinder were measured using strain gauge based force transducers, specially designed for this purpose. The force transducer essentially consists of a bulk head and a cantilever beam with provision for a strain gauge beam. At each end of the cylinder, two bulk heads of inner diameter of the cylinder were cemented rigidly to the pipe, about 120 mm from the ends of the cylinder. The cantilever beams carrying the full bridged form of strain gauges were in turn rigidly fitted to the bulk heads. The two strain gauge beams were aligned orthogonal to each other at the two ends of the cylinder so that the strain gauges in the vertical plane respond only to inline forces and the ones in the horizontal plane respond only to the transverse forces. The ends of the cylinder were properly sealed to prevent entry of water into the cylinder. The cantilever beams which slightly protrude outside were made to rest on self aligning bearings at the ends of the cylinder. These bearings were housed inside the supporting channels at both the ends of the test section. These supporting channels which ran over the whole height of the wave tank along the side walls provided the
facility for fixing the cylinder at any desired elevation from the tank floor. In order to doubly ensure any predetermined gap between the cylinder and tank bed, the channels were made to rest on the wooden blocks (cut to the required gap) at both the ends. A 2 mm clearance was maintained between the ends of the cylinder and the supporting channels.

A resistance type wave gauge fixed near the middle of the test section was used to measure the instantaneous water surface elevation above the axis of the cylinder. The cylinder was subjected to monochromatic waves of heights ranging between 30 mm and 140 mm and periods from 0.8 s to 1.2 s. The tests were carried out for various clearances of the pipeline above the tank floor, giving the gap ratios, $e/2a$ of 0, 0.05, 0.1, 0.25, 0.5 and 1.0. Three water depths of 0.525 m, 0.45 m and 0.375 m were used in the experiments for all the $e/2a$ values except for $e/2a = 1.0$ for which 0.60 m was used instead of 0.375 m. For each combination of $e$ and $d$, 30 test runs were made. This resulted in a total of 540 test runs and almost all the data were used in the analysis.

A high speed HP6942A data acquisition system was used to acquire the time histories of water surface elevation and the corresponding total inline and transverse forces. The data were collected at the sampling speed of 36000 μs/channel and 256 data samples were collected for each realization. The data thus acquired were stored in diskettes for future analysis. Sufficient time was allowed for any oscillations in the wave tank to subside after acquiring each set of data and before making the next run. The calibrations of the force transducers and wave gauge were checked from time to time.

4 Analysis and discussion of results

The analysis of the present experimental investigation was carried out based on certain assumptions. Linear wave theory was assumed to be valid to compute the water particle kinematics at the centre line of the pipe. Wave reflections from the beach and side walls were assumed to have negligible effects on the force and water surface elevation measurements. The authors have noticed in another set of experiments on velocity measurements under simulated random waves using the same test facility and ultrasonic currentmeter that the horizontal plane transverse water particle velocity was almost zero substantiating that the waves generated in the wave tank were perfectly two dimensional in the vertical plane. The ranges of different parameters that were simulated in the present experimental investigation are furnished in Table 1.

Table 1. Parameters covered in the present experimental investigation

<table>
<thead>
<tr>
<th>gap ratio</th>
<th>depth parameter</th>
<th>period parameter</th>
<th>relative pipe diameter</th>
<th>wave steepness</th>
<th>relative wave height</th>
<th>relative water depth</th>
</tr>
</thead>
<tbody>
<tr>
<td>$e/2a$</td>
<td>$d/a$</td>
<td>$K$</td>
<td>$2a/L$</td>
<td>$k/2a$</td>
<td>$H/L$</td>
<td>$d/L$</td>
</tr>
<tr>
<td>0-1.0</td>
<td>5-7</td>
<td>0.05-1.25</td>
<td>0.07-0.15</td>
<td>0.2-0.5</td>
<td>0.01-0.05</td>
<td>0.1-0.8</td>
</tr>
</tbody>
</table>

It is well known that when $K$ is very small, the flow around the pipe does not separate and the flow can be considered as potential flow around the cylinder. It is also recognized that in flows starting from rest or oscillating sinusoidally about a submerged cylinder, separation does not occur so long as the particle displacement is small in comparison with the characteristic dimension of the cylinder. Under such flow conditions, the inertia forces will dominate the other forces and the pipe is considered to be in the inertia dominated regime. The table also supports the assumption that the linear wave theory can be used to compute the water particle kinematics as the wave steepness is small.
Data correspond to:

\[ \frac{c}{2a} = 0.25 \]
\[ (d/a = 7, 6 \text{ and } 5) \]

and

\[ \frac{e}{2a} = 1.0 \]
\[ (d/a = 8, 7 \text{ and } 6) \]

Fig. 2. Validity of various wave force regimes [3].

Validité des divers régimes de force due à la houle.

Fig. 2 shows the validity of different force regimes [3] in which the typical data corresponding to \( e/2a = 0.25 \) and 1.0 and all the \( d/a \) values investigated are superimposed. Similar plots were obtained for other clearances also. From these plots it is evident that all the data in the present experimental investigation lie in the overlapping region of Morison equation and diffraction effects (inertia dominated regime) and hence any effects due to vortex generation and wake formation will be negligibly small and can be conveniently ignored. The ranges of all the parameters, \( K, 2a/L \) and \( H/2a \) provided in Table 1 support the fact that the submarine pipeline under these conditions is in the inertia dominated regime and that the forces can be modelled as inertia forces. However, for a pipeline fixed near a seabed there will be asymmetry of flow between the pipeline and the seabed and above the pipeline. This causes the asymmetry of pressure distribution about the horizontal plane through the axis of the pipe and induces a vertical lift force on the pipeline.

Thus the inline force on the pipeline is modelled for the inertia force as in equation (4). The transverse force is considered to be a linear combination of vertical lift force and vertical inertia force as in equation (5). The inline hydrodynamic coefficient of inertia, \( C_m \) and transverse hydrodynamic coefficients of lift, \( C_L \) and vertical inertia, \( C_{mv} \) are evaluated utilizing the measured forces on the model pipeline and through the use of least squares method.

The hydrodynamic coefficients in the present model investigations can be shown to be functions of the following parameters:

\[
[C_m, C_L, C_{mv}] = f(ka, e/2a, d/a)
\]  

in which

\[ d/a = \text{depth parameter} \]
\[ ka = \text{scattering parameter} \]
\[ k = \text{wave number} \]

An attempt is made herein to correlate these hydrodynamic coefficients with the wave and pipeline parameters.
4.1 Data analysis

In data reduction, middle third of about 5 to 6 waves in the record and the corresponding inline and transverse forces are considered for analysis. A general assessment of the present data is considered necessary before pursuing further with the data processing. Two of the typical measured data of waves and the corresponding inline and transverse forces for two extreme wave periods and larger wave heights used in this study are shown in Fig. 3. From this figure, it is observed that the maximum inline forces occur around the wave phase angles of maximum inline water particle acceleration (zero crossings) implying that the inline forces are dominated by the acceleration forces. On the other hand, the maximum transverse forces occur around the wave phase angles of maximum $u$ and $w$ (crest and trough of a wave) indicating that the transverse forces are dominated by the vertical lift and vertical inertia forces. An examination of all the present data showed similar variation of inline and transverse forces and supported the fact that the data are well contained in the inertia dominated regime. It is seen from Table 1 that the present data cover the $d/L$ range of 0.18 to 0.51 ($gT^2/d = 11.96$ to 37.67). In this range of $d/L$ values, which covers nearly intermediate to deep water conditions, the inline forces tended to be sinusoidal and quite symmetrical. The transverse forces also showed similar trend with the positive and negative force maxima being more or less the same.

4.2 Variation of $C_m$ with $d/a$ and $e/2a$

The variation of $C_m$ with $ka$ for three $e/2a$ values and all the $d/a$ values investigated is depicted in Fig. 4. It is observed that $C_m$ shows no noticeable trend with the depth parameter, $d/a$ and scattering parameter, $ka$ for all the $e/2a$ values investigated. This result is in accordance with the potential flow results [7, 15], in which $C_m$ is found to be independent of $d$. The authors [1, 5] have earlier reported the results of their experimental investigation on a pipeline model subjected both to regular and random waves in the inertia-drag regime ($K = 0.5$ to 28) and found that the inline hydrodynamic coefficients, $C_m$ and $C_D$ decrease with decrease in $d/a$ and with increase in $K$. This result of decreasing coefficients with $K$ for a given $d/a$ is as expected since the pipe was in the inertia-drag regime, in which the vortex generation and wake formation will definitely have their influence on the flow conditions around the pipe and hence on the coefficients. However in the present investigation, the forces on the pipe are predominantly inertia and wake effects, if any, will
have negligible effects on the force. As the variation of $C_m$ for a given $ka$ is small at least within the present range of $d/a$ values investigated, polynomial regression of various orders ranging between 1 and 3 was attempted to fit the whole range of data using least squares method for a given $e/2a$ value. Linear regression was found to fit the data quite well with the average correlation coefficient of 0.96 and standard deviation of 0.55. These linear regression lines are also superimposed in their corresponding plots. Even in the subsequent plots involving the variation of the hydrodynamic coefficient with $ka$, only linear regression lines are drawn since they were found to give the best correlation with the data.

Fig. 5 depicts the variation of $C_m$ with $ka$ for $d/a = 1$ and all the $e/2a$ values investigated. For the sake of clarity only the linear regression lines for each $e/2a$ values are presented in this plot. Unlike in the previous plot, this plot clearly shows the trend with $e/2a$ that as $e/2a$ increases, $C_m$ decreases. This result is as expected and is in accordance with the potential flow theory results. This trend of decreasing $C_m$ with increasing $e/2a$ is due to the bottom boundary effect such that the force gets reduced as the pipe is moved away from the seabed. It is also noted that $C_m$ does not show much variation with $ka$ for any given $e/2a$ value and the mean $C_m$ for any $e/2a$ seems to approach its corresponding potential flow value at low values of $ka$. It has been determined analytically by Garrison [2] that the added mass coefficient, $V_{mA}$ for the semi-infinite potential flow past a cylinder in contact with the rigid boundary is given as $2.29 (C_m = 1 + C_{mA})$. A comparison of the present mean $C_m$ values with the potential flow results will be discussed in the later part of this section.
4.3 Phase angle of maximum inline force occurrence, $\theta_{H_{\max}}$

The variation of $\theta_{H_{\max}}$ with $ka$ for $e/2a = 0.25$ and all the $d/a$ values and for $d/a = 7$ and all the $e/2a$ values investigated is shown in Figs. 6a and 6b respectively. From both these subplots, it is noted that $\theta_{H_{\max}}$ shows no correlation either with $ka$ or $d/a$ or $e/2a$. This implies that $\theta_{H_{\max}}$ is independent of $d$, $e$ and also the flow conditions at least within the present range of $ka$ values. It can also be seen that the mean value of $\theta_{H_{\max}}$ for the overall variation in the whole range of the parameters is about $-90^\circ$ (zero crossing of wave). In Figs. 7a and 7b, the variation of $\delta$ with $ka$ is depicted respectively for the above mentioned values of $e/2a$ and $d/a$. As in the previous plots, $\delta$ is also found to be independent of $d/a$, $e/2a$ and $ka$. It is also observed that the mean value of $\delta$ is around 0°. This indicates that the maximum line force, occurs at phase angles around zero crossings. The Figs. 6 and 7 along with Fig. 3 support the basic assumption of the study and emphasize the fact that the flow around the pipeline near the seabed is potential flow and the forces are predominantly inertial in this range of parameters investigated.

4.4 Comparison of measured and computed inline forces

A comparison between the measured and computed inline force time histories for one cycle is
presented for the typical cases in Fig. 8. It is seen that the agreement between the measured and computed forces is very good. Thus it is quite clear that the inline force model, considering only the inertial term of the Morison equation as given by equation (4) predicts quite well the inline forces. Even the small difference between the measured and computed forces can be attributed to the negligible drag effects.

The degree of correlation between the measured and computed forces is in general expressed in terms of the correlation coefficient, $r$ and is defined as,

$$r = \sqrt{1 - \frac{(F_c - F_m)^2}{F_c^2}}$$

(13)

where $F_m$, $F_c$ = measured and computed forces respectively and the over bar indicates expectation of the argument. After analysing all the present data, equation (4) is found to give an average $r$ value of about 0.95 with the measured inline forces.

4.5 Variation of $C_{mv}$ with $d/a$ and $e/2a$

Fig. 9 depicts the variation of $C_{mv}$ with $ka$ for three values of $e/2a$ and all the $d/a$ values investigated. As in the case of $C_m$ for inline force, it is observed in these plots also that $C_{mv}$ shows no variation with $ka$ at least within the investigated range of $ka$. The linear regression line was
found to fit the data with the average $r$ value of 0.96 and standard deviation of 0.6. This result is also in accordance with the potential flow theory that the coefficients do not depend on water depth.

The variation of $C_{mv}$ with $ka$ and various $e/2a$ values investigated is shown in Fig. 10 for $d/a = 7$. Only the best fit lines are drawn in this figure for the sake of clarity. It is observed that $C_{mv}$ decreases with increase in $e/2a$ for a given $ka$ value. This can be attributed to the effect of the bottom boundary which gets reduced as the pipe is moved away from the boundary. It is also noted that the mean value of $C_{mv}$ for the range of $ka$ and for a given $e/2a$, is very close in magnitude with $C_m$, which will be discussed later. Nath and Yamamoto [9] also reported that the added mass coefficient for a uniform ambient flow, that is accelerating either parallel or normal to the plane boundary will be the same as for a cylinder accelerating in the same direction in a still fluid.

4.6 Variation of $C_L$ with $d/a$ and $e/2a$

The variation of $C_L$ with $ka$ for three $e/2a$ values and all the $d/a$ values investigated is depicted in Fig. 11. In general, it is seen that $C_L$ increases with increase in $ka$ for low values of $e/2a$ and does not show any trend with $ka$ for larger values of $e/2a$. This may be attributed to the difference in the flow conditions between pipe and bottom boundary and above the pipe. However, it is noted that the variation of $C_L$ with $d/a$ is insignificant, at least within the present range of $d/a$ investigated. This can be explained by the fact that under potential flow conditions, the lift phenomenon does not depend on $d$ as in the cases of both $C_m$ and $C_{mv}$.

Fig. 12 depicts the variation of $C_L$ with $ka$ and different values of $e/2a$ and $d/a = 7$. It is quite evi-
dent from this plot that $C_L$ decreases (in magnitude) with increase in $e/2a$ for a given $ka$ value. This is due to the proximity effect of the bottom boundary. It has been discussed earlier that when a pipeline is placed near a bottom boundary, there will be asymmetry of pressure distribution around the pipe as there will be increased velocity of flow between the pipe and bottom boundary than above the pipe. As the pipe is moved away from the boundary, the flow around the pipe tends to be more or less symmetrical and hence the lift force will be very small. It can also be seen in
this plot that for $e/2a > 0.25$, $C_L$ can be considered to be independent of $e/2a$ though the variation above this $e/2a$ is very small.

4.7 Phase angle of maximum transverse force occurrence, $\theta_{V_{\text{max}}}$

The variation of $\theta_{V_{\text{max}}}$ with $ka$ is shown in Fig. 13 as a typical plot for $e/2a = 0.25$ and all the $d/a$ values investigated. As in the case of $\theta_{H_{\text{max}}}$, $\theta_{V_{\text{max}}}$ shows no correlation either with $e/2a$ or $d/a$ or $ka$. It appears that the mean value of $\theta_{V_{\text{max}}}$ is slightly above 180° and this can be attributed to the slight nonlinearity of the waves. However the mean value of $\theta_{V_{\text{max}}}$ for the overall variation in the whole range can be considered to be 180° (trough of the wave) for all practical purposes as the force model (equation (5)), which predicts the maximum positive force at $\theta_{V_{\text{max}}} = 180°$.

4.8 Comparison of measured and computed transverse forces

Fig. 14 shows the comparison of measured and computed transverse force time histories for one cycle for two typical cases. The computed forces using equation (6) are found to compare favourably with the measured forces. After analysing all the data, the computed forces are found to give $r$ value of about 0.8 to 0.92 with the measured forces. It has been reported earlier by the authors.
Fig. 14. Comparison of measured and computed $F_v$ for two $ka$ values.
Comparaison des valeurs de $F_v$ mesurées et calculées pour deux cas de $ka$.

Fig. 15. Comparison of present results with potential flow theory.
Comparaison des résultats de la présente étude avec la théorie en l'écoulement potentiel.
that none of the existing transverse force formulations considered for analysis predicted the measured forces for a pipeline in the inertia-drag regime. This is attributed to the complexities of the lift force phenomenon as mentioned in section 2.2. However, in the present study, the pipe is in the predominantly inertia regime and hence the prediction of equation (5) is found to be exceptionally good for the present data.

4.9 Comparison of present and potential flow results
In sections 4.2 and 4.5, it has been mentioned that the mean values of $C_m$ and $C_{mv}$ are closer to their potential flow values for various $e/2a$ values. In order to compare the present results with the potential flow theory, the theoretical curve which shows the variation of $C_m$ (or $C_{mv}$) with $e/2a$ is drawn as given by equation (6) using the first 40 images in Fig. 15a. The experimentally evaluated data points in this plot correspond to average values for two lower $ka$ values investigated. It is quite evident that the present results are in very good agreement with the theoretical predictions. In addition, this plot shows that both $C_m$ and $C_{mv}$ decrease with increase in $e/2a$, which is as expected and compares favourably with the other existing results [11, 14, 16] also. This plot further implies the validity of the present data for the potential flow calculations. Fig. 15b shows the variation of $C_L$ with $e/2a$, as defined by equation (7). It is observed that the measured values agree favourably with the theory. Even the difference can be explained by the fact that equations (6) and (7) are derived assuming that the cylinder is moving either parallel or normal to the plane boundary in still fluid. However, in the present study, the pipe is in the wave field, in which the orbital motion of the water particles will have definite influence on the forces, especially on the lift phenomenon. It is to be noted that Yamamoto et al. [16] obtained $C_L$ by some other technique unlike in the present study by least squares method.

5 Conclusions
Regular wave induced forces on a smooth horizontal pipeline placed near a seabed were investigated experimentally in the inertia dominated regime. The inline hydrodynamic coefficient of inertia, $C_m$ and the transverse hydrodynamic coefficients of lift, $C_L$ and vertical inertia, $C_{mv}$ were correlated with the scattering parameter, $ka$, gap ratio, $e/2a$ and depth parameter, $d/a$. The results of the present study are summarized as follows:
The inline and transverse inertia coefficients, $C_m$ and $C_{mv}$ respectively are independent of $ka$ for given values of $e/2a$ and $d/a$, at least within the range of $ka$ values investigated.
The inline and transverse inertia coefficients $C_m$ and $C_{mv}$ decrease with increase in $e/2a$ for a given $ka$ value.
The lift coefficient, $C_L$ increases with increase in $ka$ for $e/2a$ upto 0.25 and beyond this value $C_L$ is independent of $ka$. Furthermore $C_L$ is independent of $d/a$ for a given $e/2a$ value.
The inline and transverse force time histories are predicted quite well by simple force models given by equations (4) and (5) respectively under potential flow conditions.
The agreement between the present results and potential flow predictions is found to be quite good.

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Notations

- $A$: inline water particle displacement
- $a$: radius of pipe
- $C_D$: drag coefficient
- $C_L$: lift coefficient
- $C_m$: inline inertia coefficient
- $C_{mv}$: vertical inertia coefficient
- $D$: diameter of pipe $(2a)$
- $d$: still water depth
- $e$: vertical gap between pipe and seabed
- $K$: Keulegan-Carpenter number or period parameter
- $k$: wave number
- $s$: distance of pipe centre from the seabed
- $t$: time variable
- $U_{max}$: amplitude of inline water particle velocity
- $u, \dot{u}$: inline water particle velocity and acceleration respectively
- $\dot{w}$: transverse water particle acceleration
- $\rho$: mass density of water
- $\sigma$: wave angular frequency
- $\theta$: wave phase angle

References / Bibliographie


